Chapter-1 Rational Numbers
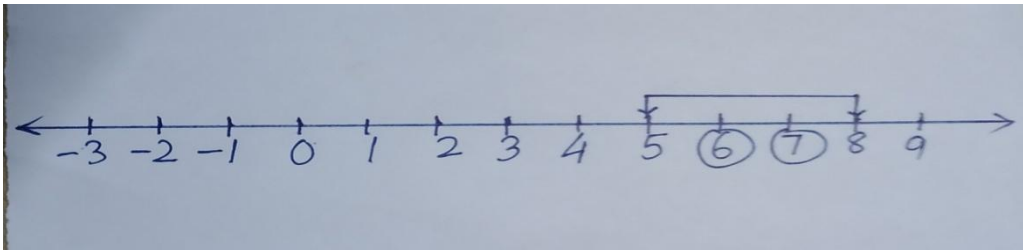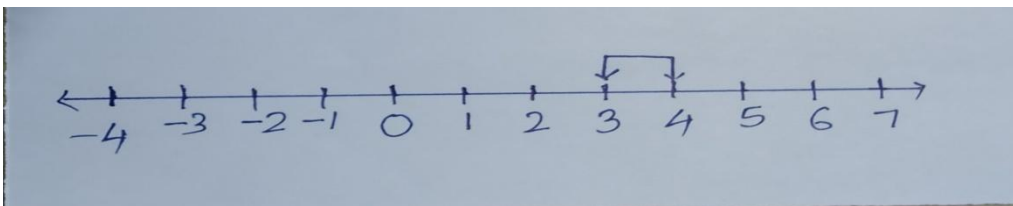
Module 4

1.4 Rational Numbers between Two Rational Numbers

Can you say how many natural numbers are there between 5 and 8?
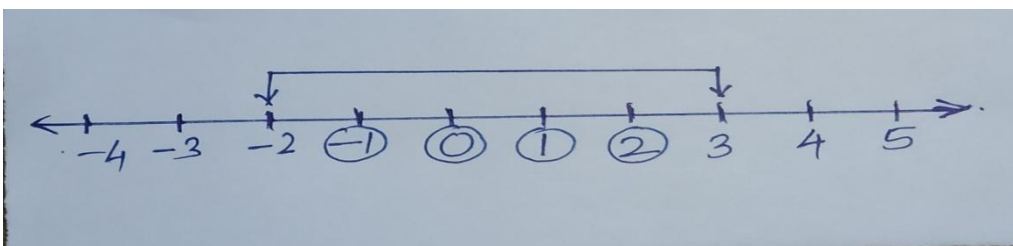

➤ Two. They are 6 and 7.

Can you say how many natural numbers are there between 3 and 4?

➤ No number.

Now how many integers are there between -2 and 3?

➤ Four. They are $-1, 0, 1, 2$

From this discussion we can say that:

There is finite number of natural numbers between any two natural numbers.

This is also true for integers.

We always have finite numbers of integers between any two integers.

Now, let us look for rational numbers.

How many rational numbers are there between $\frac{2}{7}$ *and* $\frac{6}{7}$?

You probably will think the answer is $\frac{3}{7}, \frac{4}{7}$ *and* $\frac{5}{7}$.

Now, is it correct to state that there are 3 rational numbers between $\frac{2}{7}$ *and* $\frac{6}{7}$?

➢ No, this is not the correct statement.

Let us understand the result.

$\frac{2}{7} = \frac{2 \times 10}{7 \times 10} = \frac{20}{70}$ *and* $\frac{6}{7} = \frac{6 \times 10}{7 \times 10} = \frac{60}{70}$.

The numbers $\frac{21}{70}, \frac{22}{70}, \frac{23}{70}$ ... till $\frac{59}{70}$ fall between $\frac{2}{7}$ *and* $\frac{6}{7}$.

Now as done above $\frac{2}{7}$ *and* $\frac{6}{7}$ can be written as:

$\frac{2}{7} = \frac{2 \times 100}{7 \times 100} = \frac{200}{700}$ *and* $\frac{6}{7} = \frac{6 \times 100}{7 \times 100} = \frac{600}{700}$ respectively.

Therefore you can observe unlike natural numbers and integers *there are infinite numbers of rational number between any two rational numbers.*

Q. Find any two rational numbers between 3 and 5.

Solution: $3 = \frac{30}{10}$ *and* $5 = \frac{50}{10}$.

Thus, we have $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \dots . \frac{49}{10}$ between $\frac{30}{10}$ *and* $\frac{50}{10}$ or we can say between 3 and 5.
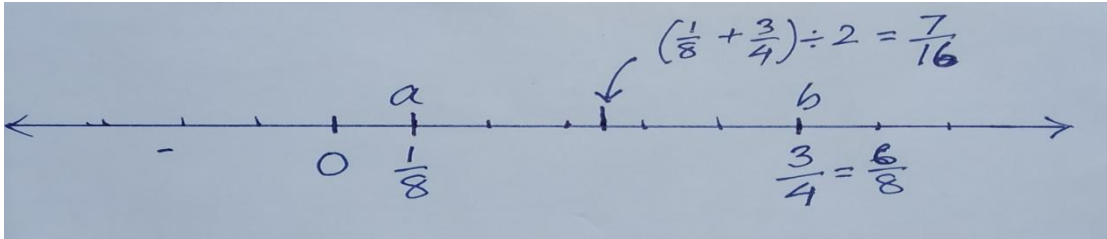
Any two rational numbers between 3 and 5 are: $\frac{31}{10}$ *and* $\frac{32}{10}$

Another method:

Let us take an example of alternative method

Q. Find a rational number between $\frac{1}{8}$ and $\frac{3}{4}$.

Solution: To find a rational number between two rational numbers by alternative method we will find the mean of two given numbers.

 For this first we will find the sum of $\frac{1}{8}$ and $\frac{3}{4}$.

The sum is: $\frac{1}{8} + \frac{3}{4} = \frac{1+6}{8} = \frac{7}{8}$

Now divide the sum by 2

$\frac{7}{8} \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$

Symbolically, we can write for any two rational numbers a and b such that $a < b$ then,

$$a < \left(\frac{a + b}{2}\right) < b$$

That is, $\frac{a+b}{2}$ lies between a and b.The rational number $\frac{a+b}{2}$ is called the mean of a and b.

Let us consider another example.

Q. Find any two rational numbers between $\frac{3}{4}$ and $\frac{5}{3}$.

As the denominator of these two numbers are different. We will convert them as rational numbers having the same denominator.

 The LCM of these denominators $4 \ and \ 3 = 12$.

Therefore, we will convert the denominator of both fractions equal to 12.

$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \ and \ \frac{5}{3} = \frac{5 \times 4}{3 \times 4} = \frac{20}{12}$.

As you can see the equivalent rational number have numerator 9 and 20 respectively.

Now the rational between them will be $\frac{10}{12}, \frac{11}{12}, \frac{12}{12}, \dots till \ \frac{19}{12}$.

So, any two rational numbers between $\frac{9}{12} \ and \frac{20}{12}$ are: $\frac{11}{12} \ and \ \frac{19}{12}$.

RECAPETULATION OF THE LESSION

- ➢ Rational numbers are closed under addition, subtraction and multiplication.
- ➢ The operation addition and multiplication are commutative and associative for rational numbers.
- ➢ Zero is called the additive identity for rational numbers.
- ➢ One is called the multiplicative identity for rational numbers.
- ➢ The additive inverse of a rational number$\frac{p}{q} \ is -\frac{p}{q}$.
- ➢ For all rational numbers a,b and c:

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

This is called the distributivity of rational number
- ➢ There are infinite number of rational numbers between any two rational numbers.

Now you can solve Exercise 1.2 of your NCERT Text book.

THANK YOU